

# Mathematical Pattern of Plasmon Surface Selection Rules According to Drude Model

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**Abstract**-This paper is concentrated on the discussion of the interaction of electromagnetic near – field with the metal surfaces. The classical Maxwell's equations are taken in considerations. The plasmonic generated from the bombardment of Zinc Oxide nanomaterial surface is the main target of the work. This can be determined using the Drude model derived from the Maxwell's equations.

**Keywords**-Plasmon, collective electronic excitation, EM wave interaction with metal surfaces. Elementary excitation, phonons, plasmons, excitations (bound electron – hole pair), Drude – Model.

## I. INTRODUCTION

The study of plasmonic is a brunch of Nanophonics. It studies the electromagnetic (EM) field can be confined over a dimension of the order or smaller than the wavelength. When the EM field interacts with the conductive electrons at the metal interface or in metallic nanostructure an enhanced optical near field of sub-wavelength is achieved [1]. The plasmon can be defined as a quantum of plasma oscillation [1] or a plasmon is a collective oscillation of the conduction electrons [2]. According to Drude – Lorentz – Sommerfeld model, the influence of external forces is considered for one electron alone and then the response is multiplied by the number of electrons. All electrons act in phase in this model [3].

## II. THEORY OF PLASMON

Some properties of laser radiation make it very interesting for tasks of nanofabrication. The radiation can be efficiently transmitted through solid, gaseous and liquid media and its energy can be easily concentrated in almost any point by the use of focusing optical elements. When focused on a solid target, intense laser radiation can cause the melting and then the ablation of the material from the target [2]. It had

already been pointed out by Pines and Bohm that the long-range nature of the Coulomb interaction between valence electrons in metals yields collective plasma oscillations similar to the electron-density oscillations observed by Tonks and Langmuir in electrical discharges in gases, thereby explaining early experiments by Ruthemann and Lang on the bombardment of thin metallic films by fast electrons [3]. According to the elementary excitations, the polaritons can be defined as coupled state between an elementary excitation and photon. While the Plasmon polaritons coupled state between a plasmon and a photon. Phonon polariton coupled state between a phonon and a photon. The excitation of a plasmon requires the interaction of an electron passing through a thin metal or reflecting an electron or photon from the metallic film. In fig.1, an incident electron interacting with a thin metal film [4]. Some properties of laser radiation make it very interesting for tasks of nanofabrication. The radiation can be efficiently transmitted through solid, gaseous and liquid media and its energy can be easily concentrated in almost any point by the use of focusing optical elements. When focused on a solid target, intense laser radiation can cause the melting and then the ablation of the material from the target, leading to the ejection of atoms and nanoclusters. The further nanocluster growth is determined by both laser parameters (wavelength, pulse length, fluence) and properties of the ambient medium [4-5]. Here are the following possibilities:

1) A pure laser ablation when the plasma is transparent to laser radiation. In this case, the radiation ablates the target and thus produces nanoclusters, which then cool down through their interaction with ambient environment (Fig. 1a). If the nanoclusters are produced in a gaseous environment, they can then be deposited

on a substrate forming a thin nanostructured film. As another possibility, the ablation in liquids leads to the release of material to the liquid and the formation of colloidal nanoparticle solution.

2) The production of laser-assisted breakdown plasma, which strongly absorbs the incoming radiation and reduces the radiation-related ablation efficiency (Fig. 1b). In general, conditions for the breakdown plasma production exist in both gaseous and liquid environment. In particular, we recently showed that this hot plasma can be used to heat the material of a semiconductor target and thus transform its properties [4].

### III. SELECTION RULES OF PLASMONS

Using the macroscopic Maxwell's equations[5] as:

$$\left. \begin{aligned} \vec{\nabla} \cdot \vec{D} &= \rho_{ext} & \dots a \\ \vec{\nabla} \cdot \vec{B} &= 0 & \dots b \\ \vec{\nabla} \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t} & \dots c \\ \vec{\nabla} \times \vec{H} &= \vec{J}_{ext} + \frac{\partial \vec{D}}{\partial t} & \dots e \end{aligned} \right\} \dots(1)$$

Where,

Eq. (1-a) refers to the electric charge is the source of the electric field.

Eq. (1-b) there is no source of magnetic field.

Eq. (1-c) change of magnetic field

In these equations there are four macroscopic fields  $\vec{D}$  the dielectric displacements,  $\vec{E}$  the electric field,  $\vec{H}$  the magnetic flux density. In addition  $\rho_{ext}$  is the external charge density, while  $\vec{J}_{ext}$  is the external current density. The total charge and current densities are written as a sum of the internal and external components [6], which are as:

$$\left. \begin{aligned} \rho_{total} &= \rho_{ext} + \rho_{int} \\ J_{total} &= J_{int} + J_{ext} \end{aligned} \right\} \dots(2)$$

Now let's define the fields,

$$D = \epsilon_0 \vec{E} + \vec{P} = \epsilon \epsilon_0 \vec{E} \quad (\vec{P} \text{ is the polarization, } \epsilon \text{ is the dielectric function})$$

$$\vec{H} = \frac{1}{\mu_0} \vec{B} - \vec{M} \quad (\vec{M} \text{ is the magnetization})$$

If we assume there is no magnetic field, thus we should concentrate only on the electric polarization effects.

$\vec{P}$  Describes the sum of the electric dipole moments per unit volume.

- 1- electron in  $\vec{E}$  - field is displaced by a distance  $\vec{r}$  and leads to  $\vec{\mu} = -e \cdot \vec{r}$ .
- For n - electrons in a volume V leads to  $n \cdot \vec{\mu} = \vec{P}$  [3,5]

Taking into account last two equations we found the relationship between the polarization  $\vec{P}$  and internal charge density:

$$\vec{\nabla} \cdot \vec{P} = -\rho_{int} \quad \dots (3)$$

Introducing this last equation into displaced equation above we obtain:

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho_{tot}}{\epsilon_0} \quad \dots (4)$$

### IV. IMPORTANCE OF DETERMINATION THE MAGNITUDE OF PLASMON FREQUENCY

According to the historical sequence of the researches, it is clear that the plasmon frequency can be determined according to equation of motion of free electron with mass m and charge e in presence of time varying electric field  $E = E(t)$ , using Coulomb's law[6]:

$$m \frac{d^2 x}{dt^2} = eE \text{ where } E = E_0 e^{i\omega t} \text{ and } x = x_0 e^{i\omega t} \quad \dots(5)$$

Or

$$x_0 = -\frac{eE_0}{m\omega^2} \quad \dots(6)$$

Thus the polarization reads

$$\vec{P} = -\frac{ne^2}{m\omega^2} \vec{E} \quad \dots(7)$$

Where (n) refers to the refraction index as a function of frequency ( $\omega$ ).

Now according to the above it is possible to compute the electric displacement field as:

$$\vec{D} = \left(1 - 4\pi \frac{ne^2}{m\omega^2}\right) \vec{E} \quad \dots(8)$$

Since we are use no external charge ( $\epsilon = 0$ ), yielding no absence of external electric field  $E = 0$ . The ( $\epsilon = 0$ ) determines therefore the condition for which

self-sustaining polarization waves can exist at the frequency. Plasma frequency can be given by:

$$(\omega_p = \sqrt{\frac{4\pi ne^2}{m}})$$

For free electrons it is possible to compute

$$\varepsilon = 1 - \frac{\omega_p^2}{\omega^2} \quad \dots(9)$$

This is also known as *Drude* model.

Fig. 1 shows the plasmon wave propagation in a TiO<sub>2</sub>nano-solid pellet. Fig 1 (a) shows that the plasmon appears as white points and some sparkles generated through the experiment of bombardment of the pellet by laser pulse. Fig. 1(b) shows plasmon propagation due to the laser pulse which relative with plasma frequency.



(a)



(b)

Fig. 1 Plasmon generation in ZnO nano-solid pellet

## V. COMPUTATION OF DIELECTRIC CONSTANT OF ZNONANO PELLET USING PLASMON CALCULATIONS

Since equation (9) is applicable to the state under test, then:

$$\varepsilon = 1 - \frac{\omega_p^2}{\omega^2}$$

The distance (d) between atoms in solid is approximately twice to the Bohr radius or:

$$d = 2a_o = \frac{2\hbar^2}{me^2} \quad \dots(10)$$

While the FCC lattice containing four atoms per unit cell

$$a = \sqrt{2} d = 2\sqrt{2}a_o. \quad \dots(11)$$

Then the electron density is:

$$n = \frac{4}{a^3} = \frac{\sqrt{2}}{d^3} = \frac{\sqrt{2}}{8a_o^3} = \frac{m^3 e^6 \sqrt{2}}{8\hbar^6} \quad \dots(12)$$

Then the plasma frequency can be given by:

$$\omega_p^2 = \frac{4\pi ne^2}{m} = \frac{4\pi e^2 m^3 e^6 \sqrt{2}}{m 8\hbar^6} = \frac{\pi m^2 e^8}{\sqrt{2}\hbar^6} \quad \dots(13)$$

Thus the plasma energy can be given by:

$$E_p = \hbar\omega_p = \hbar \sqrt{\frac{\pi m^2 e^8}{\sqrt{2}\hbar^6}} = \frac{me^4}{\hbar^2} \sqrt{\frac{\pi}{\sqrt{2}}} \quad \dots(14)$$

Fig.3 shows the experiment set-up which is used in this research. The vacuum chamber in use is designed and implemented at the laboratories of the University of Technology, Applied Sciences Dept. under our supervision.

The vacuum chamber also firstly designed as shown in fig. 2. The (gate number 1) is used as a cross KF connection where the Ar gas flow from this gate. While (gate number 3) is used as a feed through connection to feed the Langmuir probe inside the chamber. The additional welded connections are used as a view point for event capturing purposes with 45° with the axis of the cylinder vacuum chamber.

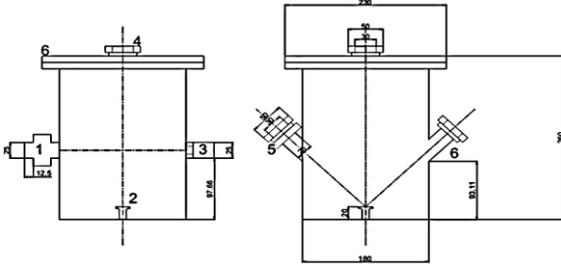


Fig. 2 Plasma vacuum chamber design



Fig. 3 Experiment set-up used to study the ZnO nanomaterial as a disc.

## VI. RESULTS AND DISCUSSIONS

Using equation (14) above, then the plasma energy can be given by:

$$E_p = \frac{me^4}{\hbar^2} \sqrt{\frac{\pi}{\sqrt{2}}} = \hbar\omega_p \text{ thus for } \omega_p = 1.542 \times 10^{15} \text{ rad/sec}$$

which is calculated depending on Langmuir probe I-V characteristics for ZnO nanomaterial which is as shown in figure (4) below.

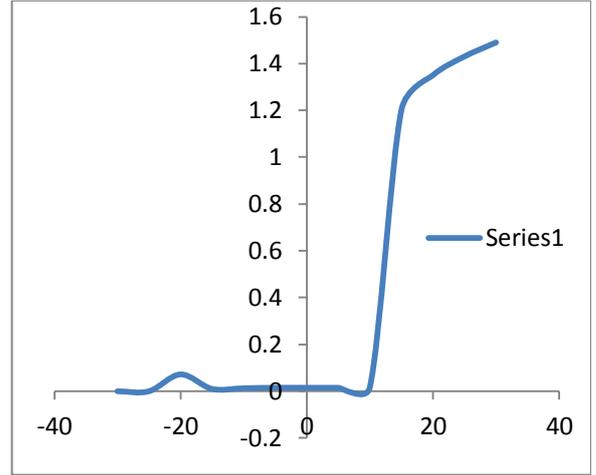


Fig. 4 The I-V characteristics of the Langmuir probe used in the experiment

$$E_p = 2.59 \times 10^{-19} \text{ J.s}$$

This is the plasmon energy according to Drude Model.

It is possible to compute the energy at different conditions since the plasmon is in motion, where this state gives a sign that the energy distribution of laser pulse on the material surface is the main reason for the plasmon movement.

Another important point of view that is the plasmon energy is smaller than the laser wavelength in use. Nd:YAG laser with  $\lambda = 1064 \text{ nm}$  or  $\lambda = 1064 \times 10^{-9} \text{ m}$ , this means the laser which is used in the photoetching process consider the plasma as an opaque material. Then it may be reflected according to this.

## VII. CONCLUSIONS

According to the probe I-V characteristics shown in fig. 4 above then it is possible to notice that there's no ion current, this is because the probe is biased with the positive potential and the drifting velocity is occurred only for the electrons to the probe. Another important point that there is no external electric or magnetic field applied to the system even the system is made from the stainless steel material. The main purpose which preventing us to applied an external electric field is not to study the plasma characteristics but the plasmon itself; beside this determines therefore the condition for which self-sustaining polarization waves can exist at the frequency.

The plasmon energy is calculated according to the Drude model which depends directly on Maxwell's equations.

#### VIII. REFERENCES

- [1] A. Zayats, *et. al.*, "Surface Plasmon Polaritons (SPPs) Introduction and basic properties", *Physics Reports* 408, 131-414 (2005).
- [2] J. Pendry, "Playing tricks with light", *Science* 285, 1687 (1999).
- [3] J. M. Pitarke and *et.al.*, "Theory of surface plasmons and surface-plasmonpolaritons," *Materia Kondentsatuaren Fisika Saila, Zientzi Fakultatea, Euskal Herriko Unibertsitatea*, 644 Posta kutxatila, E-48080 Bilbo, Basque Country, Spain, Institute of physics publishing, 2007.
- [4] Andrei V. Kabashin and *et.al.* "Laser-assisted methods for nanofabrication", *Laser Processing Laboratory, Department of Engineering Physics, Ecole Polytechnique de Montreal, Case Postale 6079, Succ. Centre-ville, Montreal, Quebec, Canada, H3C 3A7.2004.*
- [5] A. Zayats, I. Smolyaninov *et.al.*, *Journal of Optics "A: Pure and Applied Optics"* 5, S16 (2003). Overview articles on plasmonics.
- [6] M.H. March and M.P. Tosi, *Adv. in Physics* 44, 299 (1995)
- [7] Georgios Veronis, and *et.al.* "Metal-Dielectric-Metal plasmonic waveguide devices for manipulating light at the nanoscale", *Department of Electrical and Computer Engineering and Center for Computation and Technology, Louisiana State University, Baton Rouge, LA 70803, USA, Received December 23, 2008.*